

More on Universal Superconformal Attractors

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We define a general class of superconformal inflationary attractor models [1, 2], which include, among others, inflationary models of ‘induced gravity’ which were argued to retain perturbative unitarity up to the Planck scale [3]. The choice of the function $f(\phi)$ in superconformal attractors for the models introduced in [3] is $f(\phi) = \phi^n - \xi^{-1}$. We present both superconformal and supergravity versions of these models, which were derived in [1, 2] for arbitrary $f(\phi)$, together with the universal conditions required for stabilization of the extra three moduli present in the superconformal attractors, in addition to the inflaton.

The universal inflationary attractors [1, 2] are defined by one arbitrary function of a single real scalar field, $\Omega(\phi)$. This class of models in Jordan frame can be represented as follows:

$$\frac{1}{\sqrt{-g}} \mathcal{L}_J = \frac{1}{2} \Omega(\phi) R - \frac{1}{2} (\partial\phi)^2 - \frac{\lambda^2}{\xi^2} (\Omega(\phi) - 1)^2. \quad (1)$$

In the Einstein frame this model is

$$\begin{aligned} \frac{1}{\sqrt{-g}} \mathcal{L}_E = & \frac{1}{2} R - \frac{1}{2} \left(\Omega(\phi)^{-1} + \frac{3}{2} (\log \Omega(\phi))^2 \right) (\partial\phi)^2 \\ & - \frac{\lambda^2 (\Omega(\phi) - 1)^2}{\xi^2 \Omega(\phi)^2}. \end{aligned} \quad (2)$$

The superconformal generalization of these models is defined by the Kähler potential of the embedding space and the superpotential, which both depend on an arbitrary holomorphic function $\Omega(\Phi)$ [2]. The bosonic model in (1) and (2) follows from the superconformal model [2] and from the corresponding supergravity model presented in [1], upon stabilization of the extra non-inflaton scalars. The corresponding stability analysis was performed in [1] for an arbitrary function $\Omega(\Phi)$, and is valid for any choice of $\Omega(\Phi)$, as long as the slow-roll parameters during inflation are small. We will describe below the universal supersymmetric attractor models of [1, 2]. As we will see, this simultaneously provides a superconformal and supergravity embedding of the bosonic model studied in [3].

A class of models¹ called ‘induced gravity’ studied in [3] corresponds to a choice

$$\Omega_{\text{ind}}(\phi) = \xi f_{\text{ind}}(\phi). \quad (3)$$

Here $f_{\text{ind}}(\phi)$ is what was called $f(\phi)$ in [3]; we will keep the notation $f(\phi)$ for the function introduced in [1, 2]. In

refs. [1, 2] the Jordan frame model (1) was introduced for an arbitrary function $\Omega(\phi)$ and was shown to lead to an Einstein frame action (2). Particular examples of $\Omega(\phi)$ studied in [1, 2] were of the form

$$\Omega(\phi) = 1 + \xi f(\phi), \quad (4)$$

where $f^2(\phi)$ was a ξ -independent polynomial function ϕ^{2n} , or $1 + \cos(\phi/c)$. In this way our superconformal examples in [1, 2] allowed us to study simple supersymmetric models interpolating between a large set of chaotic inflation models (or natural inflation models) at $\xi = 0$ and the corresponding attractors points in the (n_s, r) -plane for increasing values of ξ , representing a non-minimal coupling to gravity $\xi f(\phi) R$. However, the general superconformal/supergravity construction developed in [1, 2] allows arbitrary $\Omega(\phi)$ and therefore arbitrary dependence of $f(\phi)$ on ξ .

To see how all induced gravity models studied in [3] are described in the context of general models developed in [1, 2] defined in eqs. (1) and (2), we have to identify the functions $\Omega_{\text{ind}}(\phi)$ with $\Omega(\phi)$

$$\Omega_{\text{ind}}(\phi) = \Omega(\phi) \quad \Rightarrow \quad \xi f_{\text{ind}}(\phi) = 1 + \xi f(\phi), \quad (5)$$

which shows that

$$f(\phi) = f_{\text{ind}}(\phi) - \xi^{-1}. \quad (6)$$

Examples of induced gravity with $f_{\text{ind}}(\phi) = \phi^n$ studied in [3] are given by our equations (1), (2) and (4) under condition that

$$f(\phi) = \phi^n - \xi^{-1} \quad (7)$$

for all values of n . It was suggested in [3] that induced gravity models and universal attractor models belong to the same class of models only for $n = 1$.² This is indeed the case in the simplest examples with ξ -independent functions $f(\phi)$ discussed in [1]. In such case our relation given in eq. (7) is not possible. However, as we already

¹ The class of models presented in [3] was argued to be unitarity-safe. The issue of perturbative unitarity violation is somewhat controversial, see, for example, a discussion in [4]. Here we will not discuss it and just show that all models of induced gravity studied in [3] can be embedded in the general class of the universal superconformal attractor models of [1, 2].

² A related observation about the unitarity-safe universal attractor models with $\Omega(\phi) = 1 + \xi\phi$ was made earlier in [5].

explained, the superconformal and supergravity models underlying eqs. (1) and (2) are defined for arbitrary function $\Omega(\phi)$. Therefore, the condition of ξ -independence of $f(\phi)$ is not necessary, which allows to compare these models by identifying their functions $\Omega(\phi)$ and leads to eq. (7) for any n . All induced gravity models described in sec. 5 of [3] are defined in our equations (1), (2), (4), (6), which explains the embedding of [3] in the more general class of models introduced in [1, 2].

A generic superconformal version of the models introduced in [1, 2] is defined by the Kähler potential of the embedding space, see [2]:

$$\mathcal{N}(X, \bar{X}) = -\frac{1}{2}|X^0|^2 [\Omega(\phi) + \Omega(\bar{\phi})] + |\Phi|^2 + |S|^2 - \frac{1}{12}|X^0|^2 [\phi^2 + \bar{\phi}^2] - 3\zeta \frac{(S\bar{S})^2}{|X^0|^2 [\Omega(\phi) + \Omega(\bar{\phi})]}, \quad (8)$$

where we are using the following notation for the complex field which has a zero conformal weight under local Weyl transformations: $\phi \equiv \sqrt{6} \frac{\Phi}{X^0}$. Here $\Omega(\phi)$ is a holomorphic function, and the superpotential is

$$\mathcal{W} = \frac{\lambda}{\xi} \frac{(X^0)^2}{3} S (\Omega(\phi) - 1). \quad (9)$$

This model in the Einstein frame leads to supergravity with $X^0 = \sqrt{3}$ and $\phi = \sqrt{2}\Phi$ defined by the following Kähler potential and superpotential [1, 2]:

$$K = -3 \log \left[\frac{1}{2}(\Omega(\phi) + \Omega(\bar{\phi})) - \frac{1}{3}S\bar{S} + \frac{1}{12}(\phi - \bar{\phi})^2 + \zeta \frac{(S\bar{S})^2}{\Omega(\phi) + \Omega(\bar{\phi})} \right], \quad W = \frac{\lambda}{\xi} S (\Omega(\phi) - 1). \quad (10)$$

This leads exactly to the bosonic model (1) discussed in [1] upon identifying the real part of the Φ -field with the inflaton, $\Phi = \bar{\Phi} = \phi/\sqrt{2}$, while $S = 0$, which is a consistent truncation. During inflation the masses of the non-inflaton fields are $m_{\text{Im } \Phi}^2 = (4/3 + 2\epsilon - \eta)V$, $m_S^2 = (-2/3 + 6\zeta + \epsilon)V$, where ϵ and η are the slow-roll

parameter, computed in case of arbitrary functions Ω [1]. Up to slow-roll corrections, with the choice $\zeta > 1/9$, all non-inflaton fields are heavy, they have masses $m^2 > H^2$ and quickly reach their minima at $S = \Phi - \bar{\Phi} = 0$, meanwhile the inflaton ϕ is light, $m_\phi^2 = \eta V \ll V$.

The minimum of the potential is a supersymmetric extremum with $DW = W = 0$ at $S = \Phi - \bar{\Phi} = 0$. At this minimum, in the class of models with $f(\phi) = f_{\text{ind}}(\phi) - \xi^{-1}$ one should have

$$\Omega(\phi) = \Omega(\bar{\phi}) = 1 \quad \Rightarrow \quad f(\phi) = f_{\text{ind}}(\phi) - \xi^{-1} = 0. \quad (11)$$

In such models $\phi = \xi^{-1/n}$ in the supersymmetric minimum, in agreement with the property of a bosonic model studied in [3].

In conclusion we would like to stress here that the superconformal attractors have an important universality property, first discovered in the models of conformal inflation in [6], where the values of n_s and r during inflation are independent on the choice of the potential $V(\varphi)$, which can be a rather general function $f^2(\tanh \varphi)$, for a canonical field φ . The analogous kind of universality we have seen in strong coupling attractors and α -attractors in [1]. The models of induced gravity studied in [3] also exhibit an attractor behavior since n_s and r during inflation are independent on the choice of n in the function $f_{\text{ind}}(\phi) = \phi^n$. As we have shown here, these models can be easily embedded in the general class of superconformal attractor models [1, 2] presented here in eqs.(1), (2), (4). This immediately provides the bosonic induced gravity models studied in [3] with a consistent supersymmetric generalization presented above. It is based on the stability analysis performed for generic superconformal attractors with an arbitrary function $\Omega(\phi)$ in [1], which valid for all models of this class where slow-roll inflation is possible.

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